THE EFFECTS OF GEOMETRY ON THE CORONA-TO- STREAMER DISCHARGE TRANSITION

An Undergraduate Research Scholars Thesis

by

KELLI DENISE HUMBIRD

Submitted to the Office of Undergraduate Research
Texas A&M University
in partial fulfillment of the requirements for the designation as

UNDERGRADUATE RESEARCH SCHOLAR

Approved by
Research Advisor: Dr. William Marlow

May 2013

Major: Nuclear Engineering
Physics

TABLE OF CONTENTS

ABSTRACT

The Effects of Geometry on the Corona-to-Streamer Discharge Transition. (May 2013)

Kelli Denise Humbird
Department of Nuclear Engineering
Texas A&M University

Research Advisor: Dr. William Marlow
Department of Nuclear Engineering

The electric spark discharge has been studied for hundreds of years, yet many details of the phenomenon remain elusive. One particular area in the field of spark discharges that has yet to be explored in depth is the transition region between the corona and the streamer discharge. The parameters that characterize the transition region are purely geometric for a given potential difference applied between two electrodes. For the case of a point-to-plane electrode geometry, the transition between the oscillating corona discharge and the rapidly-growing streamer discharge is determined by the radius of curvature of the anode. In this contribution, the transition radius of curvature is found analytically using simplified models of each discharge and the principle of least action. For a sufficiently small anode, the corona discharge is also shown to be energetically more favorable at all radii of curvature, supporting the general claim that corona discharges are most readily produced on thin wires.

ACKNOWLEDGMENTS

Acknowledgements must be made to my research advisor, Dr. William Marlow, who was of tremendous assistance in the completion of this project. The skills acquired throughout my research experience under his guidance will be of great utility in my future.

CHAPTER I

INTRODUCTION

The application of spark discharges has a history that long exceeds the theoretical description of ionization. In recent years discharges have played a crucial role in the development of new technologies, from chemical synthesis to plasma screens.[1] Despite the wide usage of electrical discharges, much of the physics governing the phenomena remains elusive.

There are various forms of spark discharges that are characterized by their degree and rate of ionization, all of which are initiated by the same mechanism that was first described by John Townsend in 1897.[2] Townsend's collision theory of ionization states that a free electron accelerated by a sufficiently strong electric field can transfer enough energy to a gas molecule upon collision to dislodge one of the molecule's electrons. This newly freed electron is then accelerated by the electric field, and can go on to free an additional electron, leading to what is called an ionization cascade or a Townsend avalanche. The free electron population, n_e, increases exponentially in time according to Eq. (1):

$$n_e = n_0 e^{\nu t} \tag{1}$$

where n_0 is the initial number of free electrons and ν is the experimentally determined ionization frequency, the average number of ionization events per electron per unit time.[2] The exponential increase in the electron population is countered by the recombination of free electrons with positive ions and electron attachment to neutral molecules.[3] If the electric field is sufficiently strong, the ionization cascade dominates the electron loss mechanisms and continues until it reaches an oppositely charged electrode, leading to the phenomenon known as the electric arc. In a weaker field, the avalanche comes to a halt before the gap between the electrodes has been bridged.

The rate and degree of ionization are affected by several parameters such as the electric potential, electrode geometry, and gas properties. Manipulating these parameters results in various gas discharges tailored for specific uses. The discharges of concern in this paper are the corona and the streamer discharge.

The corona discharge

The corona is a low energy discharge that can be made to occur in several situations; it can form on either a positive or negative electrode, and can be manually pulsed or self-repeating. For a discussion of the different types of coronas, see Veldhuizen's "Corona discharges: fundamentals and diagnostics" [3] or Goldman's "The corona discharge, its properties and specific uses" [1].

The particular corona discussed here is a positive, self-repetitive discharge. "Positive" refers to the polarity of the smaller electrode on which the corona forms; this electrode is traditionally labeled the anode. "Self-repetitive" indicates that the discharge is DC-driven; the characteristic oscillations of the corona are self-induced, not forced by an AC source. To create the non-uniform electric field, a "point-to-plane" electrode configuration is used; the setup consists of a thin wire anode placed with its axis perpendicular to a large grounded plane. The discharge is initiated by free electrons within the vicinity of the anode tip. The origins of these electrons are not of concern to us; they could be present from the background radiation, introduced using an external electron source, or thermally emitted from the highly charged anode.[3,4] Once in the vicinity of the anode, the electron is accelerated toward the wire and undergoes collisions with surrounding gas molecules as it travels. In a sufficiently strong electric field, the electron gains enough energy to ionize the gas, giving rise to the familiar Townsend avalanche. In the case of the corona, the avalanche ends well before it can close the gap between the anode and cathode.

The newly freed electrons are highly mobile and quickly make their way toward the anode surface, forming a thin layer of negative charge around the wire. This leaves the almost-stationary positive ions in a large cloud surrounding the negative charge. This area of space charge development is called the ionization region, recognizable by its faint glow, while beyond the space charge is the non-glowing diffusive region.[1] Eventually the electric field induced by the formation of the positive space charge cancels the applied field within the ionization region and the avalanche stops. Diffusion, recombination and electron attachment processes take over in the absence of electric forces until the gas is almost entirely neutral. When the applied electric field becomes dominant again, a remaining free electron is accelerated back toward the anode, initiating a new ionization cascade. The oscillations of the net electric field within the ionization region are self-sustaining.[4]

Corona discharges are studied not only because of their interesting physical characteristics, but because of their important applications in industry. One particular specialized use of the corona discharge is the chemical synthesis of ozone.[5] The corona method of ozone production is the most affordable and requires only atmospheric air to produce O_3 concentrations of 3-6%. Coronas are also commonly used to induce chemical reactions for surface treatments to enhance particular properties.[1] For example, the treatment of polymers to increase adhesiveness is important for printers and photo-copying, while the reduction of electric charge on an aircraft surface is essential to prevent uncontrolled discharges during flight that could severely hinder aircraft performance.[6]

The streamer discharge

A streamer discharge can develop on the same point-to-plane electrode configuration used to illustrate the corona; however, it requires a higher electric field and is not an oscillatory process. As with any other discharge, the streamer begins with a Townsend avalanche, but unlike the corona, the cascade is not suppressed by the development of positive space charge. The higher electric field increases the free electron energy to the point that the "electron temperature", the temperature defined by the kinetic energy of the electrons alone, increases significantly. Near the anode where the ionization cascade is most concentrated, the temperature is high enough to cause an increase in the electrical conductivity of the gas and a plasma region forms.[2,7] One of the most important phenomena associated with a plasma is called Debye shielding: in the presence of an electric field the plasma will become polarized so that the charge distribution reduces the field within the plasma.[8] The plasma region near the anode effectively extends the equipotential surface of the wire further into the inter-electrode gap by forming a highly conductive path along the trail of the ionization cascade.[2] Consequently, the high electric field region that began at the tip of the wire is relocated to the end of the ionization path, also known as the streamer head. Free electrons in this new high-field region induce further ionization cascades, allowing the streamer to propagate toward the cathode.

The streamer is cylindrical in shape; the positive ions in the surrounding region provide Coulombic forces that act to focus the negatively charged channel. After inception, the streamer often branches at the head as it travels into lower field regions and may either stop entirely before reaching the cathode, or continue until it crosses the electrode gap, forming an electric arc.[7] What happens to the streamer after its inception is another rich area of research beyond the scope of this paper. For discussions on streamer propagation, see references 2 and 9.

Streamers are the early formations of a spark discharge, so their applications are as far-reaching as the application of sparks. Regardless of their specific use, they are crucial to understand to ensure safety in electrical devices where sparks can cause extreme damage.

CHAPTER II

METHODS

Corona and streamer discharges are often studied independently for specific applications, so the experimental conditions under which each particular discharge can be produced are well-known. However, there has not been much discussion of the transition between the two discharges, despite the fact that streamers often evolve from coronas.[2,10] The transition region may be of importance in applications, such as when a streamer discharge needs to be produced with as low of a voltage as possible, or a high energy corona is required but streamer development must be avoided.

The purpose of this paper is to show the possibility of quantifying the corona-to-streamer transition based on purely geometric considerations using simplified models of the two discharges. The transition is expected to be determined almost exclusively by the electric field strength. As previously discussed, the distinguishing feature of the corona discharge is the cancellation of the applied electric field by the induced field of the positive space charge. As the applied electric field is increased, it becomes too strong for the positive space charge to cancel, at which point the self-sustaining corona discharge is no longer stable and the ionization cascade leads to a streamer formation.

For point-to-plane electrodes held at a fixed potential difference, the electric field is altered by varying the radius of curvature of the tip of the wire anode. As the radius of curvature is decreased the electric field strength increases, and at some maximum field strength the streamer discharge is expected to prevail over the corona. This transition is quantified using the principle of least action, which states that natural processes occur in a way such as to minimize the total energy of the system. In the case of the corona versus streamer, this implies that the discharge that requires the least amount of energy is the most probable. The energy of each discharge is calculated for a variable radius of curvature of the anode tip and the discharge that is most likely to occur is predicted via the requirement of minimal energy.

An approximate energy of each discharge is calculated analytically using simplified models of the streamer and the corona. The streamer model used for these calculations is illustrated in Figure 1.

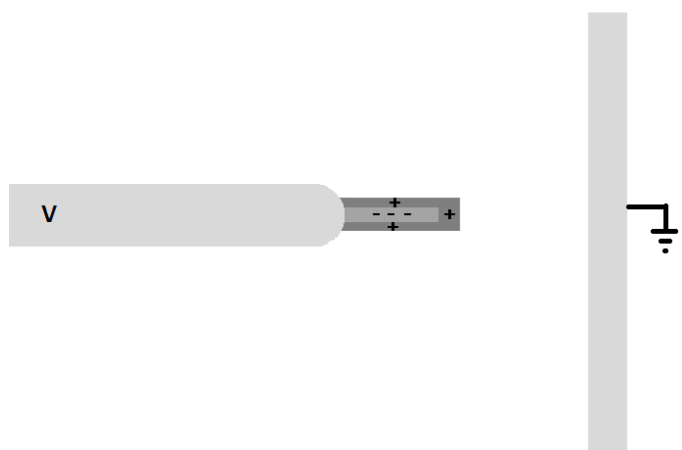

Figure 1. Streamer model for point-to-plane electrode configuration.

The point at which a streamer attaches to the anode before expanding into a cylindrical channel of nearly-constant diameter covers an area on the order of a few square microns.[3,10] This area is a fraction of the area of the anode tip, which for a one millimeter diameter wire is on the order of a square millimeter, therefore the radius of curvature of the anode tip has a negligible effect on the streamer attachment geometry. It is therefore assumed that the streamer channel size and therefore the energy are independent of the radius of curvature. This idea has been validated by previous experiments, in which the streamer channel radius was shown to be roughly constant, independent of the field or electrode.[2,3,10] Given these justified assumptions, only one streamer energy calculation is required to describe the system for a fixed applied potential difference. The corona energy, however, does depend on the anode radius of curvature. For point-to-plane electrode configurations, the corona discharge is typically modeled as a set of concentric spherical shells of charge surrounding the anode tip.[11] The thickness of the shells for a particular applied voltage is not altered much as the anode radius of curvature varies, but the shell radii correspond directly to the radius of the tip on which they form. This model is illustrated in Figure 2.

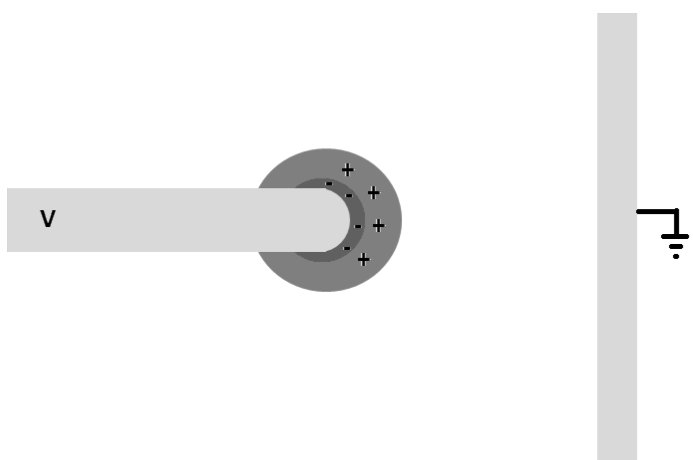

Figure 2. Corona model for point-to-plane electrode configuration.

The total energy of each discharge is calculated in the limit that the length scales are much larger than the electron and ion mean free paths so that the kinetics of individual gas molecules can be neglected in favor of their collective, average behavior. In this limit, the potentially significant contributions to the total energy are those due to the electric field strength, the magnetic field strength, and the kinetic energy associated with the flow of electrons and ions. The electromagnetic field energy is given by Eq. (2) with the integrals taken over all space:

$$U_{EM} = \frac{\varepsilon_0}{2}\int E^2 d\tau + \frac{1}{2\mu_0}\int B^2 d\tau \tag{2}$$

where ε_0 and μ_0 are the permittivity and permeability of free space; the relative permittivity and permeability for air having been taken to be unity. The kinetic energy due to the motion of the electrons and ions is non-relativistic and is approximated by Eq. (3):

$$K = N \cdot \frac{1}{2} m\bar{v}^2 \tag{3}$$

where $\bar{v}$ is the average electron velocity of the electron/ion, *m* is the electron/ion mass, and *N* is the total number of electrons or ions.

The energy required to sustain the electrodes at their fixed potential difference is identical in both the streamer and corona calculations, and therefore does not need to be included. The addition of this energy simply increases all calculated energies by a constant amount, and therefore has no effect on the determination of the prevailing discharge.

With the energies of each discharge known, the principle of least action dictates which discharge is favored for certain radii of curvature and the critical radius at which the transition takes place can be determined.

CHAPTER III

RESULTS

Gas discharges are difficult to describe by a purely theoretical approach; therefore much of the data used for the following calculations was borrowed from the results of previous experiments, including those from references 7, 12, and 13. Consequently, there are errors and inconsistencies associated with experimental procedures that carried into the energy calculations. Fortunately, knowledge of the precise values of parameters such as charge densities and gas velocities are not essential to illustrate the general characteristics of this model. Orders of magnitude of such variables are consistent throughout multiple experiments, giving confidence that the values selected for the energy calculations will reflect reality to a good approximation. Although this hypothesis is tested using a specific set of experimentally determined parameters, the overall trend is expected to be the same for any carefully obtained set of data.

Corona calculations

The anode wire is chosen to have a one millimeter radius, which determines the minimum radius of curvature of the tip. To take advantage of the simplicity of spherical symmetry, the anode tip is viewed as a spherical section whose radius is equal to the radius of curvature. For the corona energy, the calculations are carried out using a full sphere and the fraction of the energy that is associated with the anode tip is found by simple geometric reasoning. A particularly straightforward way to find the fraction of the sphere that contributes to the corona energy is to take a surface area fraction, F_{SA}, as given by Eq (4).

$$F_{SA} = \frac{\iint_{\theta\prime=0,\emptyset=0}^{\theta'=\theta,\emptyset=2\pi} R^2 sin\theta\prime d\theta\prime d\emptyset}{\iint_{\theta\prime=0,\emptyset=0}^{\theta'=\pi,\emptyset=2\pi} R^2 sin\theta\prime d\theta\prime d\emptyset} = \frac{1-cos\theta}{2} \tag{4}$$

The angle θ can be expressed in terms of the radius of the wire and the radius of curvature. Figure 3 illustrates this relationship.

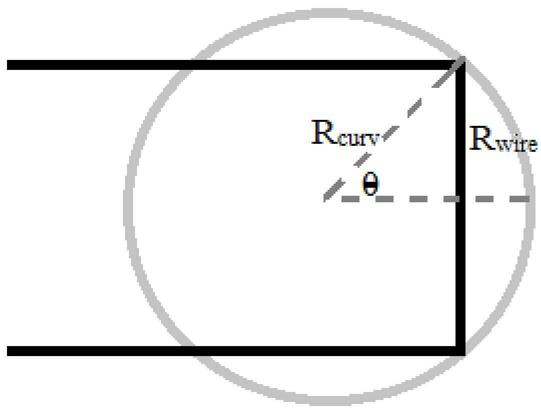

Figure 3. Relationship between the angle θ, the wire radius, and the radius of curvature.

The wire anode is the cylindrical structure whose tip is a spherical section, with radius $R_{curv.}$ It can be seen from the diagram the angle θ is given by Eq. (5):

$$\theta = \arcsin\left(\frac{R_{wire}}{R_{curv}}\right) \tag{5}$$

Once the energy for a complete spherical anode has been calculated, this value multiplied by F_{SA} yields the energy associated with the anode tip.

The corona energy is calculated assuming the quasi-static approximation; the discharge is frozen at a particular point in time, and the problem is treated as being electrostatic. Although the corona is a transient process, this approach is justifiable because of the time scale differences in the stages of the discharge. The “rise time” of the corona-the time it takes to reach the point illustrated in Figure 2 after the initial ionization- is on the order of a nanosecond, while the "decay time" for diffusion and recombination reactions to neutralize the gas is on the order of microseconds.[13] Since diffusion takes significantly longer than the time required for the corona to develop, it is assumed that once the space charge is fully developed, the charges diffuse slowly enough that they are comparatively stationary.

The magnetic field contribution to the energy is insignificant regardless of the quasi-static assumption since the current is on the order of a micro-ampere, therefore the magnetic contribution the energy is neglected without introducing substantial error.[13] It is also assumed that the kinetic energy contribution to the system energy is negligible. Although the electrons move quickly, they have such a small mass that their kinetic energy contribution is only on the order of a pico-joule, and since the ions are almost stationary throughout the process they provide little kinetic energy to the system. It follows that the corona energy is dominated by the electric field term; in the following calculations all other terms will be disregarded without introducing appreciable error.

The charge densities are assumed to be uniformly distributed; the layers of charge are thin and the electric field will not vary much across the thickness of the layer. The electric fields inside the negative and positive charge layers and outside the space charge distributions are determined by Gauss's law and the energy is calculated using Eq (2), with the magnetic term set to zero.

Streamer calculations

The streamer model takes advantage of cylindrical symmetry; the channel has a uniform negative charge density with radius R_-, and is surrounded by a coaxial, uniform shell of positive ions with radius R_+. Because the streamer channel is a plasma, care must be taken to ensure that the electric field inside this channel is calculated correctly. Plasmas redistribute their charge density in such a way to reduce external electric fields; this is called Debye shielding.[8] The distance over which the field is reduced exponentially is called the Debye length, λ_D, defined by Eq. (6)

$$\lambda_D = \sqrt{\frac{\varepsilon_0 kT}{2n_0 e^2}} \tag{6}$$

where k is Boltzmann's constant, T is the temperature in Kelvin, n_0 is the electron density, and e the electron charge. The "Debye sphere" is a spherical region with a radius equal to the Debye length. If the population of electrons within this sphere is large, plasma physics considerations must be made to correctly determine the electric field. In this situation the discharge is occurring at room temperature with a maximum electron density on the order of 10^{21} electrons/m^3, yielding a Debye length of approximately 300 Å, and less than one electron located within a Debye sphere. The charge population within the streamer is small compared to that in a plasma, therefore shielding effects are insignificant and the electric field is calculated as usual.

For reasons similar to those given for the corona discharge, the magnetic and kinetic energy terms are negligible compared to the electric field energy. Although the streamer current is higher than the corona current, the discharge is short lived and does not create a magnetic field that is strong enough to contribute an appreciable amount of energy to the system.

The streamer geometry does not lend itself to analytical calculations without making an additional assumption. The electric field of a finite cylinder with complicated boundary conditions must be solved numerically. However, if it is assumed that the contribution to the energy due to the end of the cylinder is negligible compared to the energy due to the radial component of the electric field, a completely analytic derivation of the energy is possible. The length of the streamer channel is approximately a factor of ten greater than the streamer diameter in this model, therefore the radial component of the electric field will contribute significantly more energy to the system than the field at the tip of the channel.

Results

The electron and ion densities and the volumes of the negative and positive charge regions are selected to be consistent with the results of previously reported experiments.[13] These values can be found in the appendix.

The single streamer energy and corona energies for a wide range of curvatures were calculated using Mathematica 9, a standard computation program. With a fixed anode wire radius of 1mm, the energy of the corona along with the constant streamer energy is plotted against the radius of curvature in Figure 4 on the next page.

The results illustrate that for large radii of curvature the corona discharge requires less energy than the streamer, and is therefore the favored discharge. As the radius of curvature is decreased, the corona discharge begins to require more energy until at some particular transition radius, in this case 1.08mm, the corona and streamer energies become equal. Below this transition radius the corona is no longer stable and the streamer becomes energetically more favorable.

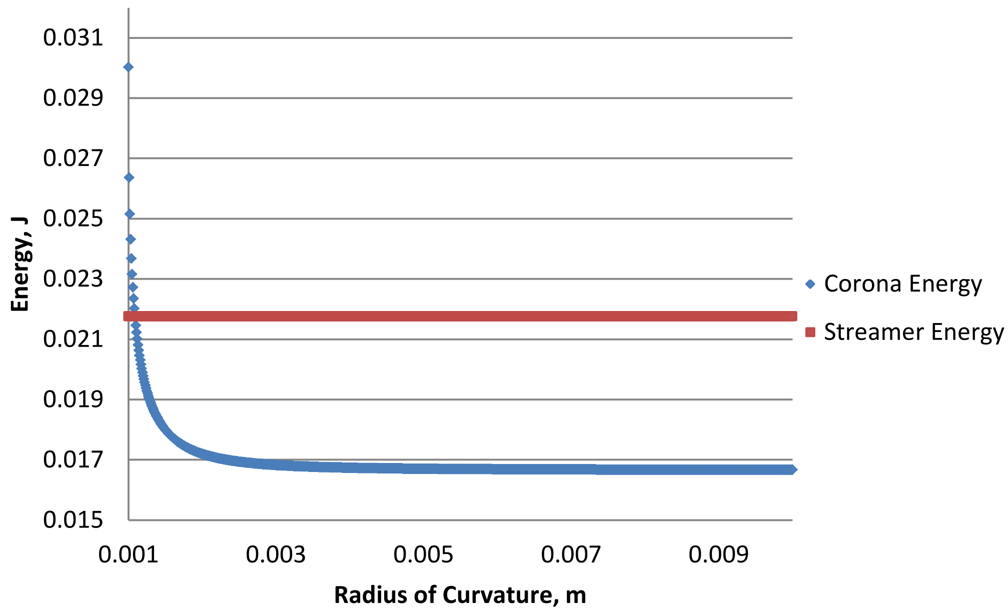

Figure 4. Streamer and corona discharge energies for various radii of curvature.

Since these calculations relied on already existing experimental results, altering the system's parameters to explore the effects of varying more than just the radius of curvature is difficult. For example, these calculations were carried out with the results of experiments where the applied voltage was 24kV, so the effect of lowering the voltage cannot easily be predicted. However, if the changes made to values used in the initial calculations are small compared to their overall magnitude, certain limits can be safely explored without concern of whether or not the model is still valid. One particular alteration that can be made to the system is a slight change of the anode wire radius.

Papers and books that discuss spark discharges often state that corona discharges predominantly occur on "thin" wires, while streamers and sparks occur on larger wires for sufficiently high applied voltages.[2,9] This begs the question: What is a “thin” wire? What is the maximum anode wire radius that can be chosen such that the corona discharge is energetically more favorable than the streamer for all curvatures?

In an attempt to quantify the meaning of “thin” the discharge energy calculations are performed for various anode wire radii until one is found such that the maximum corona energy is equivalent to the streamer energy. For the particular values chosen for this model, this occurs at a wire radius of 0.86mm; a small enough deviation from our original 1mm radius to eliminate concerns about whether or not the model is valid. This result indicates that the requirements for an electrode wire to be considered “thin” is quantifiable for a given experimental set up. For situations in which a thin wire is used, if a corona discharge is produced, it is without the risk of the corona developing into a streamer or arc. This can be of extreme value in industrial applications such as surface treatments, where corona discharges are used to initiate small scale chemical reactions and the accidental production of a high energy streamer could ruin the surface being treated.

CHAPTER IV

SUMMARY AND CONCLUSIONS

The corona and streamer electric discharges are often utilized in industry and experiments, but much of the physics that governs the phenomena is difficult to explain theoretically. Using simplified models of each discharge, it is possible to quantify certain parameters that determine when the transition from a corona to a streamer will take place. The corona is a self-sustaining discharge characterized by an oscillating net electric field, while the streamer is a non-repeating, rapidly-growing discharge. These phenomena can occur in very similar situations and it is expected that the discharge that occurs is that which requires the least amount of energy. For a fixed potential difference applied to a point-to-plane electrode configuration, the radius of curvature of the anode tip is varied and the energy of each discharge calculated. For larger radii of curvature the corona requires less energy than the streamer, and is therefore the favored discharge. As the radius of curvature is decreased, the electric field becomes too strong for the oscillating corona to remain stable, and the streamer discharge becomes energetically more favorable. When the diameter of the anode wire is allowed to vary, it is found that at a particular size wire the corona energy is less than the streamer energy for all radii of curvature, indicating that a sufficiently small wire at a particular potential difference can produce only a corona discharge; the streamer will never be energetically favored.

Knowledge of the transition between the corona and streamer discharges can be used to ensure safety in spark discharge applications. As long as one stays away from the transition region the type of discharge that will occur can be predicted with confidence.

REFERENCES

1. “The corona discharge, its properties and specific uses” M. Goldman, A. Goldman, R.S. Sigmond, Pure & Applied Chemistry Vol. 57 No. 9, (1985).

2. “Spark Discharges”, E.M. Bazelyan, Yu. P. Razier, CRC Press LLC, (1998).

3. “Corona discharges: fundamentals and diagnostics”, E.M. van Veldhuizen, W.R. Rutgers, www.phys.tue.nl, (2001).

4. “An Introduction to Nonequilibrium Plasmas at Atmospheric Pressure”, S Nijdam, E Veldhuizen, P Bruggeman, U Ebert, Wiley (2012).

5. “Corona Discharge vs. UV Ozone Generation”, Ozone Applications, http://www.ozoneapplications.com/info/cd_vs_uv.htm, (2012).

6. “What is the Corona Effect”, J. Parmar, http://electricalnotes.wordpress.com/2011/03/23/what-is-corona-effect, (2011).

7. “Ionization processes in spark discharge plasmas” N.L. Aleksandrov, E.M. Bazelyn, Plasma Sources Science & Technology, (1999).

8. “Plasmas”, R. Spencer, http://maxwell.byu.edu/~spencerr/phys442/plasma.pdf, (2012).

9. “The Mechanism of the Electric Spark”, L. Loeb, J. Meek, Stanford University Press, (1941).

10. “Pulsed positive corona streamer propagation and branching”, E M van Veldhuizen and W R Rutgers, J. Phys. D: Appl. Phys. 35 (2002).

11. “Analysis of Corona Onset Electric Field Considering the Effect of Space Charges”, T. Lu, IEEE Transactions of Magnetics Vol. 47 No. 5, (2011).

12. “Study of the Characteristics of a Streamer Discharge in Air Based on a Plasma Chemical Model”, W Sima et al, IEEE V 12, (2012).

13. “The theory of positive glow corona”, R Morrow, Journal of Physics, Applied Physics, V 30, (1997).

APPENDIX

Streamer Energy

Figure 5 illustrates the charge distribution geometry used to calculate the streamer energy.

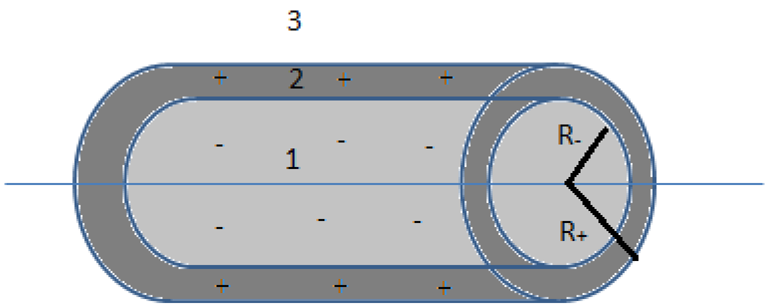

Figure 5. Streamer charge distribution.

The radial components of the electric field for the streamer discharge are found using Gauss's law in integral form:

$$\oint \vec{E} \cdot d\vec{A} = \frac{-Q_{enclosed}}{\varepsilon_0} \tag{7}$$

where E is the electric field, $Q_{enclosed}$ is the charge within the area over which the integral is taken, and ε_0 is the permittivity of free space.

The radial component of the electric field in regions 1, 2 and 3 are given as follows:

$$E_1 = \frac{\rho_- r}{2\varepsilon_0} \tag{8a}$$

$$E_2 = \frac{1}{2\varepsilon_0 r}\left[\rho_+(r^2 - R_-^2) - \rho_- R_-^2\right] \tag{8b}$$

$$E_3 = \frac{1}{2\varepsilon_0 r}\left[\rho_+(R_+^2 - R_-^2) - \rho_- R_-^2\right] \tag{8c}$$

where the radii are as labeled in the above figure, ρ_+ is the positive space charge density, and ρ_- is the negative space charge density.

The energy due to the electric field is given by the integral:

$$U = \frac{\varepsilon_0}{2}\int E^2 d\tau \tag{9}$$

Integrating the electric fields over the appropriate volumes yields the following expression for the energy of the streamer discharge due to the electric field:

$$U_{streamer} = \frac{\pi l \rho_-^2 R_-^4}{16\varepsilon_0} + \frac{\pi l}{4\varepsilon_0}\left[\rho_+(R_+^2 - R_-^2) - \rho_- R_-^2\right]^2 \ln\left(\frac{R_{max}}{R_+}\right) + \frac{\pi l}{4\varepsilon_0}\left[\rho_+\left\{\frac{1}{4}(R_+^4 - R_-^4) - R_-^2(R_+^2 - R_-^2) + R_-^4 \ln\left(\frac{R_+}{R_-}\right)\right\} - 2\rho_+\rho_- R_-^2\left\{\frac{1}{2}(R_+^2 - R_-^2) - R_-^2 \ln\left(\frac{R_+}{R_-}\right)\right\} + \rho_-^2 R_-^4 \ln\left(\frac{R_+}{R_-}\right)\right] \tag{10}$$

Corona Energy

Figure 6 illustrates the charge distribution geometry used to calculate the corona energy.

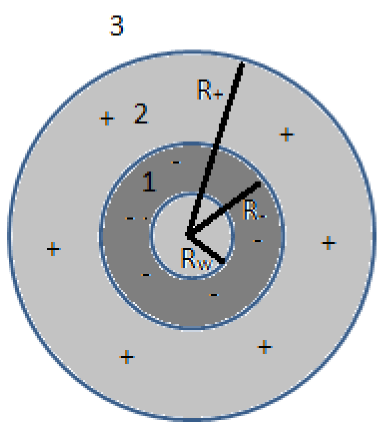

Figure 6. Corona charge distribution.

The electric field of the corona discharge in regions 1, 2, and 3 are given by equations 11a, 11b, and 11c, respectively:

$$E_1 = \frac{\rho_-}{2\varepsilon_0}\left(\frac{r^3 - R_W^3}{r^2}\right) \tag{11a}$$

$$E_2 = \frac{1}{3\varepsilon_0 r^2}\left[\rho_+(r^3 - R_-^3) - \rho_-(R_-^3 - R_W^3)\right] \tag{11b}$$

$$E_3 = \frac{1}{3\varepsilon_0 r^2}\left[\rho_+(R_+^3 - R_-^3) - \rho_-(R_-^3 - R_w^3)\right] \tag{11c}$$

The energy, as calculated using the integral in Eq. 9, is calculated using the expression:

$$U_{corona} = \frac{2\pi\rho_-^2}{9\varepsilon_0}\left[\frac{1}{5}(R_-^5 - R_w^5) - R_w^3(R_-^2 - R_w^2) - R_w^6\left(\frac{1}{R_-} - \frac{1}{R_w}\right)\right] + \frac{2\pi}{9\varepsilon_0}\left\{\rho_+^2\left[\frac{1}{5}(R_+^5 - R_-^5) - R_-^3(R_+^2 - R_-^2) - R_-^6\left(\frac{1}{R_+} - \frac{1}{R_-}\right)\right] - 2\rho_+\rho_-\left[\frac{1}{2}(R_+^2 - R_-^2)(R_-^3 - R_w^3) + (R_-^6 - R_-^3 R_w^3)\left(\frac{1}{R_+} - \frac{1}{R_-}\right)\right] - \rho_-^2(R_-^3 - R_w^3)^2\left(\frac{1}{R_+} - \frac{1}{R_-}\right)\right\} + \frac{2\pi}{9\varepsilon_0}\left[\rho_+(R_+^3 - R_-^3) - \rho_-(R_-^3 - R_w^3)\right]^2\left(\frac{1}{l} - \frac{1}{R_+}\right) \tag{12}$$

Values used for calculations

Streamer length, l : 0.002 m

Streamer channel radius, R_- : 250*10^{-6} m

Streamer positive space charge radius, R_+ : 1.25*10^{-3} m

Streamer positive space charge density, ρ_+ : 160 C/m^3

Streamer maximum radius (defines integration region), R_{max} : 0.01 m

Negative space charge density determined by conservation of charge:

$$(\rho V)_+ = (\rho V)_- \tag{13}$$

Corona negative charge region radius, R_- : (radius of curvature)+100*10^{-6} m

Corona positive charge region radius, R_+ : (radius of curvature)+1*10^{-3} m

Corona maximum radius (defines integration region), R_{max} : 0.01 m

Corona positive space charge density, ρ_+ : 160 C/m^3

Negative space charge density determined by conservation of charge as stated in Eq. (13).